NASA TECH BRIEF

Study Compares Methods for the Numerical Solution of Ordinary Differential Equations

The problem:

To determine which of the many available methods for the numerical solution of ordinary differential equations is the most efficient by evaluating the performance of each of the alternatives in solving identical problems.

The solution:

A report prepared for Marshall Space Flight Center by the Georgia Institute of Technology that compares the use of five different methods for the computer solution of the restricted three-body problem. The report, entitled "Study of Methods for the Numerical Solution of Ordinary Differential Equations," describes the implementation of each method on a Burroughs B-5000 computer and in terms of speed and accuracy.

How it's done:

The following methods for integrating nonlinear coupled differential equations were tested in the study:

a. The single-step Lie Series method
b. The multistep Cowell method
c. The multistep Adams method
d. The single-step Runge–Kutta–Fehlberg method
e. The single-step Runge–Kutta–Shanks method

Each of these methods was applied to the same set of equations, and all were programmed on the B-5000 computer to obtain a comparison in terms of speed and accuracy. The differential equations tested were the equations of motion of an earth–moon satellite using the simplifying assumptions that the earth-to-moon distance is constant and that a satellite remains in the plane of the lunar orbit. Periodic orbits are known for this system, and a check for periodicity provides a gage of accuracy for each method.

Programs for each method were written in double-precision floating-point arithmetic (23 decimal places) in Extend Algol for the B-5000 computer. A series of runs was made on 3 different Arenstorf orbits at various orders from 7 to 16 and accuracies from 10^{-12} to 10^{-16}.

Each of the five methods is discussed in detail in a separate section of the report. Each section contains an introduction, a description of the mathematical method, a discussion and an explanation of a computer program utilizing the method, and a flowchart and program listing. A summary section compares the advantages and disadvantages of each method and suggests applications for each method.

The overall conclusions reached were that each of the methods, except that of Cowell, could be considered effective, but the methods of Runge–Kutta–Shanks and Runge–Kutta–Fehlberg were the best. At the highest accuracies and orders, where Runge–Kutta–Shanks formulas are not available, the Runge–Kutta–Fehlberg method was superior.

Note:

Inquiries concerning this innovation may be directed to:

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Reference: B66-10466

Patent status:

No patent action is contemplated by NASA.

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Category 01